Languages, Litanies, and the Limit

The Discourse of Mathematics in Neal Stephenson's *Anathem*

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In this article, I explore Stephenson's use of mathematical objects and philosophies in his novel *Anathem* (2008). Comparing it to Plato's *Timaeus*, I argue that the novel should not be read as a literal expression of Stephenson's own philosophical commitments, but that it should instead be treated as a thought-experiment in metaphysical possibility. I then situate the novel in the context of mathematical philosophy, and by means of close readings of the relevant passages, proceed to argue that the conclusion of *Anathem* suggests a possible reconciliation between Platonist and Fictionalist philosophies of mathematics.

One of the most recent additions to Stephenson's literary *oeuvre*, *Anathem* is also perhaps his most ambitious. Combining world-historical narrative, science fiction gadgetry, contemporary philosophy, and characteristically wry humor, Stephenson puts together a massive novel of ideas, consisting of free interplay between philosophy, science, and story-telling, of a sort only rarely seen in contemporary science fiction. While much of the philosophical discussion within Stephenson's text focuses on the polycosmic theory of consciousness advocated in the later portions of the book, it is my intent in this article to restrict analysis of the many philosophical themes of the novel to that of mathematical realism, or the question of which mathematical objects constitute "real" entities. This limitation is both to fit this article into a manageable scope, but also to focus on those issues alone in *Anathem* which my own educational background permits me to address: while my education has included formal study in mathematics, I am in no way qualified to hold forth on issues in theoretical physics or neurophysiology. In this article I will argue that *Anathem* puts forth a dialectical reconciliation of the two dominant themes of contemporary mathematical philosophy, Platonism and Fictionalism, arguing that each represent different but mutually compatible ways of understanding the same phenomena, as a result of a metaphysics advocated by the novel which I will loosely describe as "transcendental materialism." In exploring this line of argument, I will first briefly situate *Anathem* in the context of Stephenson's own corpus, as well as outlining the thematic indebtedness of the novel to Plato's *Timaeus*. I will then discuss the mathe-

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matical-philosophical concerns salient to the content of *Anathem*, and finally analyze the various representations and discussions of mathematical philosophy over the course of the novel.

Form and Function

In the concluding paragraphs to *In the Beginning … Was the Command Line,* a piece of short non-fiction focusing in turns on the relative merits of competing operating systems, the history of the computer, and the role of the computer in society at large, Stephenson permits himself to indulge in a bit of metaphysical speculation, in which he describes the universe as a computerized operating system of cosmic proportions, run by a deistic systems administrator.

> [S]omewhere outside of and beyond our universe is an operating system, coded over incalculable spans of time by some kind of hacker-demiurge […] The demiurge sits at his teletype, pounding out one command line after another, specifying the values of fundamental constants of physics […] When he's finished typing out the command line, his right pinky hesitates above the enter key for an aeon or two, wondering what's going to happen; then down it comes—and the *whack* you hear is another big bang. (112)

As a piece of philosophy, this image is clearly problematic: not only does Stephenson naively attribute gender to a problematically transcendent being, but it is furthermore unclear just where this "somewhere" where the demiurge is coding could possibly be, if it is "beyond" the universe. Of course, this image is not intended as philosophy or metaphysics, and it is somewhat unfair to demand such a high level of rigor. Rather, it seems clear that this image is intended as a kind of poetry, a myth of the cosmos if you will, an explanation of the world that does not seek to enable predictions, but instead seeks to satisfy certain psychological needs for order, unity, and form in the world writ large. As such, both the content and the function of this concluding image from *In the Beginning … Was the Command Line*, and especially its use of the word "demiurge," recall another famous story-of-the-world, the one set forth in Plato's *Timaeus*.

The bulk of Plato's *Timaeus* presents the principal, eponymous speaker as he describes "the origin of the universe" and "the nature of human beings" (27a). Timaeus attributes the creation of the universe and of humanity to the δημιουργός or *demiurge*, often translated in English as craftsman, maker, or

fashioner, but literally meaning "public worker." The structural relation between the demiurge and physical reality in the *Timaeus* is much the same as that articulated by Stephenson: a single, powerful entity creating a universe out of a strictly aesthetic, disinterested impulse. As with Stephenson's account, however, the reader of the *Timaeus* must be careful about the extent to which she or he is to attribute the ideas and theories set forth in the *Timaeus* to Plato's own beliefs on the nature and origin of the physical universe—John M. Cooper argues that "one should exercise more than ordinary caution in inferring from what Timaeus says to details of Plato's own commitments even on matters of philosophical principle" (1225). As Timaeus himself points out, "if we can come up with accounts no less likely than any, we ought to be content," concluding that "we should accept the *likely* tale on these matters" (29c–d, emphasis added). Thus, both the conclusion of *In the Beginning … Was the Command Line* and the *Timaeus* maintain the same tenuous relation with questions of metaphysics and theology, navigating between outright fictions and formal philosophy to pursue the balance of the "likely story," not impressed by the author with the stamp of certitude, but nevertheless not implausible in its own right.

Certain readers of this article may wonder just how the preceding two paragraphs may contribute to an analysis of the philosophy of mathematics as developed in *Anathem*. It is my argument that *Anathem* holds much the same position with relation to the rest of Stephenson's *oeuvre* as the *Timaeus* holds to the rest of Plato's work. Considering each writer's respective influence on the development of, say, Western thought as a whole, the comparison may come across as a bit absurd. However, insofar as each reflects the pet, unverifiable cosmological theories of the writer, the legitimacy of the analogy becomes clear. Furthermore, *Anathem* can be read as an application of both (uncontroversially) Stephenson's metaphysics as developed in *In the Beginning … Was the Command Line* as well as (more controversially) much of the cosmological theory developed by Plato in the *Timaeus*. Of course, while Stephenson attributes the science and philosophy of *Anathem* to many contemporary and past thinkers, only off-handedly mentioning Plato in his acknowledgements (982, www.nealstephenson.com/anathem/acknow.htm), he repeatedly acknowledges the significance of Platonism to the development of the ideas, as expressed by other writers such as Roger Penrose and Hao Wang. I simply do not know whether or not Stephenson has ever read the *Timeaus*, although given the extent of his auto-didacticism it is certainly a plausible and perhaps even "likely," supposition; nor do I know if he is aware of the extensive parallels between *Anathem* and the *Timaeus*. Nevertheless, as

can be inferred from Stephenson's own acknowledgements as well as close readings of *Anathem* and the *Timaeus*, *Anathem* is clearly situated in the discursive lineage proceeding from *Timaeus*—it is situated further down the "Hylaean Theoric Flow," to put it in the metaphysical terms of *Anathem*—and consciousness of this thematic indebtedness is productive in the mathematical discussion that is to follow.

The Story

Anathem is set on the planet of Arbre, an Earth-like scene that shares many physical and biological features with Earth, but differs from its sociological conventions in several key aspects. Perhaps the most important of these is the existence on Arbre of the "mathic world," a global monastic system consisting of practitioners, called "avout," who swear devotion to the mathic creed and choose to spend their lives strictly in the mathic world, limiting their contact with the "Saecular Power" —the avout term for all non-mathic governmental entities—to certain pre-ordained observances and ceremonies. Unlike traditional monasteries, with their primary focus on theological study and religious observation, the concents of Arbre (as the individual mathic entities or monasteries are known) are predominantly rationalist and secular in their epistemology: the juxtaposition of the discourse of a modern (non-religious) university with the social arrangement of a medieval monastery comes fairly close to describing the structure of the mathic world. The mathic world can be further divided into various classes in two different ways. The first is by the length of the vows sworn by the avout to the mathic order. Each concent is constituted by one or more "maths," each of which hosts avout committed for a certain time-span in the mathic world. These maths come in one-year, ten-year, hundred-year, and thousand-year varieties, known respectively as Unarians, Decenarians, Centenarians, and Millenarians. Furthermore, avout also identify as members of faculties, analogous to monastic orders, which predicate themselves upon various epistemological and political commitments, as well as participation in certain fields of research. The most significant of these faculties to both the plot of *Anathem* and the argument of this article are the Halikaarnian and the Procian orders, which differ over fundamental views of language: the Halikaarnian or "semantic" faculty emphasizes the ability of words to "mean" independent of individual cognition, while the Procian or "syntactic" faculty emphasizes the performance side of language, arguing that words are symbols that mean only

what is attributed to them by human thought. Finally, due to vaguely referenced disastrous events of apocalyptic proportions that took place in the distant past, avout are forbidden nearly all forms of technology, excepting the bolt, the chord, and the sphere, austere elements of clothing which, due to the atomically unique nature of the material from which they are made, nevertheless admit of extreme technological value. Of course, the avout are permitted and even encouraged to study the theory of technology as much as possible; they are merely forbidden from building or even handling such technology while residing in the mathic world.

Anathem is presented as the first-person narration of Erasmas, a late-teenaged Decenarian in the Concent of St. Edhar, who is on the verge of committing to the Halikaarnian faculty with which he will identify for the remainder of his time in the mathic world, and presumably for the remainder of his life. Erasmas's initiation into the mathic world, however, is disrupted by an event of world-historical significance: the discovery of an alien spaceship orbiting Arbre, armed with sufficient nuclear weaponry to pose a threat to the political stability and autonomy of the entire planet. The Saecular Power requests the assistance of the mathic world in responding to the alien threat, and a global conference known as a Convox is called to formulate an appropriate strategy; it is revealed that the aliens are exceedingly Arbran-like in form, being in fact from a series of somewhat parallel dimensions that includes Earth. Erasmas, along with a handful of other avout and even a sympathetic member of the alien spaceship, is sent on a commando mission to destroy the nuclear weapon. The success of this mission initiates a series of peace talks between the Magisterium (as the joint venture between the Saecular Power and the mathic world is called) and the alien craft. The novel concludes with the signing of a peace treaty, and the advent of a new era of mathic–Saecular relations on Arbre.

Philosophically, *Anathem* is predominantly concerned with recent (and highly speculative) innovations in the theory of quantum consciousness. In building up this theory, the novel embraces the polycosmic interpretation of quantum theory, which holds that every possible quantum interaction describes a slightly different but nevertheless real cosmos. That is, what is possible but unactualized in the present cosmos is actualized in some other cosmos: to describe an unrealized event as a "potential" is to say that the event in question has actually happened in some other universe. These other cosmi, as differentiated from our own, come in two kinds, which the novel does not explicitly distinguish between but which I will describe as Neighboring and Platonic. Neighboring cosmi differ from each other in relatively

trivial ways, differing in only a single decision from each other. Thus, in this cosmos, I had sugar in my coffee this morning. Under the polycosmic theory of quantum interpretation, there exists a Neighboring cosmos exactly like the present cosmos, except that there I had Splenda instead. There also exist cosmi in which I had tea instead of coffee, and in which I did not have anything to drink because I overslept. The Platonic-type cosmos is perhaps more familiar to most readers; it does not differ from the present cosmos in terms of single differences, but instead transcends it in some quasi-mystical manner; hence my identification of this metaphysical system as a kind of "transcendental materialism" in the opening paragraph of this article. With this theory of physical reality in mind, the novel imagines human consciousness to be a kind of "sixth sense," which enables the individual to perceive Neighboring cosmi insofar as they represent counterfactuals from the present cosmos, as well as Platonic cosmi. It is in respect of the individual's relation to the Platonic cosmi that the bulk of my argument will focus, as this metaphysically speculative understanding of the universe enables Stephenson to present an innovative response to the Platonist–Fictionalist impasse.

The preceding summary should make clear the sociological, plot, and philosophical references internal to the text that will comprise the following analysis. Nevertheless, a further difficulty in developing critical accounts of *Anathem* is the novel's frequent use of neologisms.[1] When the context does not make the meaning of a given term sufficiently clear, I will include a footnote to the relevant entry from the dictionary included at the back of *Anathem*.

Introduction to the Philosophy of Mathematics

Professor of philosophy Andrew D. Irvine has observed that "One of the most striking features of mathematics is the fact that we are much more certain about what mathematical knowledge we have than about what mathematical knowledge is knowledge *of*" (ix). The equation known as Euler's Identity is, I think, an excellent example of this epistemic phenomenon, perhaps unique to mathematics. Named after its discoverer/inventor Leonard Euler, arguably the most dominant figure of eighteenth-century mathematics, the equation reads

$$e^{i\pi} + 1 = 0$$

As readers may recall, the irrational constant π can be defined as the ratio between the diameter and the circumference of a circle, and is roughly equal to 3.14159. Readers may further recall the imaginary constant *i*, which is equal to √-1, and the irrational but real constant *e*, which has many important applications in calculus and is roughly equal to 2.71828. Thus, Euler's Identity beautifully relates five of the most important constants in mathematics (including the numbers 1 and 0) in a simple and elegant formula. The proof of this identity is well established, and to the best of my knowledge no mathematician has ever contested the legitimacy of this expression; nevertheless, at a certain level it is not at all clear just what Euler's Identity *means* outside of contextual applications of the equation, and, to paraphrase Irvine's observation, while it is quite clear that Euler's Identity makes a true statement, it is not at all clear just what it is about which the statement of Euler's Identity is true.

I have chosen Euler's Identity as representative of the problematics of mathematical representation not because it is uniquely problematic with respect to the breadth of mathematical discourse, but instead for the surprising and defamiliarizing conjunction of its various constants, which psychologically underscores the epistemologically problematic nature of mathematical statements. Whatever ontological status Euler's Identity may have, Platonists and Fictionalists alike agree that the same is certainly true for every other mathematical statement in circulation, from the Pythagorean Theorem to the Fundamental Theorem of Calculus. It is the precise nature of this ontological status that forms the nature of the dispute between these two competing schools of thought.

From classical antiquity to the middle of the eighteenth century, the philosophy of mathematics was dominated by a school of thought known as Aristotelian Realism, which "holds that mathematics is a science of the real world, just as much as biology or sociology are" (Franklin 103). In the system of metaphysics advocated by Aristotle, entities are endowed with four "causes"—the efficient, the formal, the material, and the final. Under this schematic, mathematics becomes a branch of the study of formal causes, an analysis of the structural relations contained within and between objects and systems. Thus, to assert that "all equilateral triangles are equiangular" is to make a claim about all equilateral triangles simultaneously, and specifically about the formal cause that endues equilateral triangles with their unique structure; in a conversation between Alan Turing and Lawrence Waterhouse in the early portions of *Cryptonomicon* (2000), Stephenson describes this view of mathematics as a kind of physics of sticks and bottlecaps. This view

of mathematics began to be problematized with the development of non-physical systems of mathematics in the late eighteenth century, in which mathematicians began to prize logical coherency instead of physical reality as the determining factor of mathematical legitimacy—as Stephenson puts it, "when mathematicians began fooling around with things like the square root of negative one, and quaternions, then they were no longer dealing with things that you could translate into sticks and bottlecaps" (18). Exemplary of this epistemic shift are the development of mathematical theories of "hypercubes," or cubes existing in four dimensions. While there exists no physically observable four-dimensional cube, it is nevertheless possible to make coherent claims about structural properties of the hypercube, including the number of vertices it possesses (16) as well as the size of its "quartic" volume (length x width x height x its extensionality in the fourth dimension). It is in response to this crisis of epistemology that the schools of Platonism and Fictionalism began to gain widespread acceptance in the mathematical-philosophical community, as each proposed coherent but ostensibly exclusive solutions to the problems of what mathematics is knowledge *of*.

While competing definitions of these two schools of thought abound, and thinkers who subscribe to each of these models frequently contradict each other in profound, apparently irreconcilable ways, certain generalizations of each can nevertheless be procured. W. D. Hart has argued that "Platonism is the metaphysical view that there are non-mental, non-physical abstract objects like numbers that do not depend on us or anything mental or physical for their existence" (24). Mark Balaguer has likewise observed that "Platonism [...] can be expressed as the view that the mathematical realm is plenitudinous; in other words, the idea here is that all the mathematical objects that (logically possibly) *could* exist actually *do* exist" (40). In the other camp, Daniel Bonevac has described Fictionalism as "the view that mathematical discourse is in some respect fictional: mathematical objects [...] have the same metaphysical status as Sherlock Holmes or Macbeth" (345), while Mark Kalderon has characterized fictionalism as the "commitment [...] that acceptance in a given domain of inquiry need not be truth-normed, and that the acceptance of a sentence from an associated region of discourse need not involve belief in its content" (2). Thus, Platonist philosophies of mathematics center around the idea that mathematics is knowledge of real, metaphysical entities, however divorced they may appear to be from physical reality as we know it, while Fictionalist philosophies of mathematics emphasize the technological nature of mathematics, describing it as a game played with symbols devoid of semantic content that nevertheless enable useful

interactions with the physical world. This debate is not unlike the essentialist–constructivist debate of cultural studies, and is nearly isomorphic with the Halikaarnian–Procian disputes that feature prominently in many of the philosophical dialogues of *Anathem*.

Having completed a brief discussion of the nature of the literary function of *Anathem* as well as a survey of the relevant history of mathematical philosophy necessary to develop an understanding of the mathematical–philosophical strategies of the implied author of *Anathem*, we are now in a position to approach an analysis of the text itself.

Mathematics as Manifestation

The earliest occurrence of mathematical phenomena in *Anathem* comes in the course of descriptions of the math offered by Fraa Erasmas. These descriptions are incidental, related in a stream-of-consciousness style of narrative as Erasmas hurries across the concent to arrive on time at the clock-winding ceremony in which he is expected to take part. Nevertheless, the details, though subtle, are profound. As Fraa Jad mutters over the zipper to his newly acquired pair of jeans, "Topology is destiny" (328): the topology of the math, that is, the architecture of the math, indicates the destiny toward which the story will tend.

"The Præsidium,"[2] the reader is told:

> stood on four pillars and for most of its height was square in cross-section. Not far above the dials, however, the corners of the square floor-plan were cleaved off, making it into an octagon, and not far above that, the octagon became a sixteen-sided polygon, and above that it became round. The roof of the Præsidium was a disk, or rather a lens, as it bulged up slightly in the middle to shed rainwater. It supported the megaliths, domes, penthouses, and turrets of the starhenge, which drove, and was driven by, the same clock-works that ran the dials. (12)

The tower's structure is such as to replicate the linear construction of the circle. A circle can, with some reduction, be thought of as an infinite-sided polygon.[3] The tower's design suggests an iterative process, by which a square becomes an octagon, an octagon becomes a 16-gon, a 16-gon becomes a 32-gon, and so on, with perfect circularity as the ultimate goal. In light of the paradigmatic discourse that has surrounded the circle for nearly three millennia that associates it with the idea of perfection, this architectural

feature suggests an ambition of transcendence on the part of the concent, an attempt to construct, through discrete steps, a direct connection to a higher reality. However, the process does not stop with mere "roundness"; the Præsidium is capped with a lens, suggesting further transcendence in the motion from the circle to the sphere, which in turn supports the mechanisms by which the members of the concent can come to know the cosmos in which they are inscribed. Knowledge, it can be inferred, is the product of a reaching for the infinite. Finally, and perhaps most importantly, the Præsidium is itself a clock tower, suggesting a three-way equivalence relation between time, knowledge, and the infinite, recalling Heideggerean theories of the interrelatedness of being and time. Unfortunately a thorough analysis of this tempting theme is beyond the scope of this article.[4]

Descriptions of architecture over the following hundred or so pages continue to be salted with mathematical observations. The chancel of the Mynster[5] is described as having "the symmetry group of the eighth roots of unity" (16), a machine shop just outside the concent walls displays "rational adrokhonic[6] geometry" (84) in its roof-trusses, a rotunda inside the concent has an oculus "shaped like an isosceles triangle" (125), and the floor of another chamber in the concent is decorated with "numerous visual proofs of the [Adrakhonic Theorem]"[7] (129), all of which combine to suggest a ubiquitous, if often latent, mathematical presence in the world of *Anathem*.

Two major mathematical ideas are developed in the few hundred pages immediately following these descriptions. Although they arise naturally in the course of the story, they are relegated to the appendix due to their somewhat technical nature. The first of these so-called calci,[8] lifted almost directly from Plato's *Meno*, attempts to solve the problem of doubling-the-square. That is, it attempts to identify, given a square of arbitrary size, precisely what steps are necessary to construct a square with an area exactly twice as large. The details of the solution are not relevant here, but as a consequence two points are introduced that will prove to have immense thematic significance.

The first concerns the dialectical nature of problem solving: "It seemed impossible [to figure it out]. But by thinking it through, we were able to come up with an answer. And not just an approximate answer, but one that is perfectly correct" (960). It is implied that, given sufficient data (and in theoretical mathematics, all data at hand is sufficient nearly by definition), knowledge can be had by simply learning to ask the right questions, a technique that Orolo will emphasize to Erasmas time and again over the course of the novel.

The second point concerns the relationship between theoretical geometry

and spatio-temporal reality. Following the successful construction of the doubled square, Erasmas congratulates Dath on solving a millennia-old problem. When Dath objects that sheet cake (the mathematical modeling system used in the proof) was not around so many thousand years ago, Erasmas acknowledges the objection, but points out that other things to which the derived results apply were around, and suggests a point of inquiry. "Why should a proof that works for sheet cake work as well for a plot of land? Cake and land are different things" (148). The property of geometry that allows for such ubiquitous implementation will become a major theme of the role of mathematics in *Anathem*.

In the dialog that makes up the bulk of the second calca, Erasmas explains to the young mathic initiate Barb the nature of abstract vector spaces, which he describes as "abstract theorical spaces that follow different rules, called action principles" (205). Problem-solving techniques are not as emphasized in this second calca as they are in the first, largely due to the fact that the first calca concerns proof taking place within the context of an already axiomatized system—that is, a system in which the fundamental "rules" are already known, and argumentation proceeds by interpretation and application of these rules—whereas the second calca concerns the development and justification of an entirely new axiomatic system, or set of fundamental rules. Instead, the principle of ubiquitous application, the ability of mathematical thought to theoretically apply to any given physical situation regardless of time, place, or context, is more fully developed. In short, the calca concludes that, given a set of objects with measurable properties, it is always possible to construct a system that will concomitantly and unambiguously account for all the properties of all the objects at any given time. As developed in the calca, the claim is simplistic, non-rigorous, and does not provide much indication as to how such an account is to be developed for any particular such set of objects. Nevertheless, the implication of the guaranteed existence of such a system suggests a kind of solution to the old philosophical problem of the One and the Many:[9] though each object is distinct, by participating in the same abstract vector space, or "Hemn space" as the novel terms it, they become, in a sense, one. As before, this theme will prove to have vital significance to the philosophical development of the novel as a whole.

The Præsidium and the two calci represent strictly theoretical investigations into mathematics; the first incident involving the application of the mathematically theoretical to the physically actual takes place in a sort of basement beneath Shuf's Dowment in the concent of St. Edhar. This application concerns the attempt to pick out the mysterious object in polar orbit over

Arbre from the mess of other satellites following a similar trajectory. After obtaining a photomnemonic tablet with several month's nighttime footage, Erasmas observes a pie-cut type diagram (277–78), with each diameter representing the path of a given satellite over the north pole. Of course, mathematically speaking, the locus of the pie-cut diagram should be somewhat off-center, a little over half-way between the center and the circumference of the circle, as the concent is not located at the North Pole, as a literal interpretation of the diagram would imply. However, the important observation to make is that Erasmas is here using mathematics to accomplish a definite, practical end; mathematics, far from being an abstract, theoretical, "impractical" discourse in this text, can with a moment's notice prove to have a definite and useful purpose.

Two more mathematical diagrams in the text deserve a brief mention. On receiving the first set of classified photographs of the alien spacecraft, Erasmas observes a curious diagram on one of the faces of the ship. "It was obviously a proof," the narrator says, "almost certainly of the Adrakhonic Theorem" (360). The diagram included in the text raises two points of particular interest. On the one hand, it becomes clear that the Cousins (as the aliens are called at this point in the text) possess mathematics, and systems based on mathematical systems of reasoning. Not only that, they possess the same "kind" of mathematics as the inhabitants of Arbre, recalling Erasmas's earlier point following the original calca on doubling-the-square, that the principles of geometry are ubiquitous. At this point, theory has become demonstrated in praxis. "*Logic. Proof.* The Cousins had these—had them in common with us" (362), as Erasmas himself observes.

However, a far more curious, and somewhat more subversive, point to observe concerns the nature of the proof as supplied in the text. The three squares constructed along the edges of the central triangle, in conjunction with the several lines drawn from the vertices of the central triangle, indicate that this proof is the particular one supplied by Euclid in the original *Elements*. Yet Euclid's proof is perhaps the most mathematically inefficient of well-known proofs of the Pythagorean Theorem currently circulating in the mathematical community, lagging far behind Qurra's Proof, the anonymous, so-called "Chinese Proof," and even Garfield's Proof, published by the soon-to-be president five years before he took office.

A handful of possible explanations for this phenomenon come to mind, none of them entirely satisfactory. The Cousins could be aware of the prevalence of Euclid's proof among the Arbran community, but the age of the ship, and more significantly the age of the diagram on the ship, would seem to

contradict this presupposition. Stephenson himself could be suggesting that Euclid's proof is the most "natural" of the proofs, but the history of mathematics, with which Stephenson is surely familiar, clearly undermines the tenability of such of an account. Most likely, the solution to this problem lies in reader-response theory, as Euclid's proof is likely the most easily recognized of the Pythagorean proofs, due to an unfortunate precedent in the history of Western mathematics education, as high school geometry courses have focused almost exclusively on strictly Euclidean theories.

Finally, a brief word is appropriate on the photograph of the analemma found on the wall of Orolo's temporary residence on Bly's Butte. Of course, an analemma is the shape drawn by the sun's cotemporaneous position in the sky over the course of a year, a combined function of the earth's axial slant and elliptical orbit. The presence of the analemma here, and its use in communicating with the Cousins in later portions of the novel, suggests that the Cousins not only trivially have object recognition skills, being able to identify Orolo's analemma projected at St. Edhar (714) with the analemma inscribed at Orithena, but also share astronomical knowledge with the Arbrans. That is, not only do the Cousins share theoretical geometry, but they also share physical, or applied geometry, as well as the intellectual sophistication to navigate the semiotic gap between the two modes of discourse.

The remainder of the novel is relatively devoid of explicitly articulated mathematics. Instead, the plot focuses on the application of the mathematics developed by this point in the novel, and includes several metaphysical "reveals" about the nature of time, the cosmos, and pretty much everything else, resulting in the metaphysical picture of the world outlined previously in this article. Yet each of these reveals refers back to mathematical principles already discussed, whether quantitative principles, involving proper mathematical objects, or qualitative principles, involving techniques of mathematical argumentation. Mutually interacting as a kind of dialectical feedback loop, these principles in turn shed a profound light on to the nature of the mathematical principles in question espoused in the text.

Mathematics as Metaphysics

Before approaching the metaphysics of mathematics, however, it is appropriate to spend a brief period considering the methodologies of argumentation as presented in the text, as the significance of the philosophical presuppositions underlying the acceptance of these methodologies to the

semantic content of their conclusions cannot be overemphasized.

Running throughout the text there seems to be a tension between methodologies characterized by processes of deduction and methodologies characterized by processes of aesthetics. On several occasions, reference is made to Diax's Rake as a first principle of argumentation, and similar appeals are made to Gardan's Steelyard, the Arbran equivalent of Occam's Razor. Yet counter to these principles of unemotional, "objective" reasoning runs a subversive discourse analogous to Keats's famous dictum of beauty as truth. "Nothing is more important than that you see and love the beauty that is right in front of you" (113), Orolo tells Erasmas. Despite the dismissive responses Erasmas is sure the investigative attitude characterized by privileging beauty will bring, he observes that Saunts in the past "judged theorical proofs not logically but aesthetically" (117). Ultimately, the end product of this discourse concerning beauty seems to work to destabilize the position of empirical methodology as the sole purveyor of truth, not replacing it, but offering a methodological alternative instead. After all, as Arsibalt tells Erasmas, "If you can't test it, it's not theorics—it's metatheorics" (272). A different discipline demands a different methodology, but though the methodology might differ, it is nevertheless implied that the legitimacy of the conclusions remains the same.[10] Ultimately, it is on the basis of these truth-signifying aesthetic principles that the metaphysical system of the novel is accepted by its characters, in much the same way as the metaphysical and physical systems of the *Timaeus* are accepted by the interlocutors of the dialog.

As mentioned earlier, the majority of *Anathem*'s philosophical content concerns the development and exploration of the many-worlds interpretation of quantum phenomena, and theories of quantum consciousness based on this interpretation. The mathematics of Neighboring cosmi—those that differ from the present cosmos by a set of small, relatively insignificant differences—is picked up on in the discussion of plausible worldtracks early on in the final third of the book. The dialog in question concerns the meaning of "to happen"—"What does it mean for something to happen?" Erasmas asks his interlocutor (681). Building on the second calca concerning points in Hemn space, it is concluded that for an event to take place, it must do so within the context of a "coherent *series* of such points: a worldtrack that *could have happened*" (682). The commonsense plausibility of this claim is in effect an application of intuitively held notions of analytic continuity, which, reductively put, claim that for any two points on continuous function, there exists a "smooth" curve that connects the two points. Such a curve is indeed

constructible; in the absence of extreme restrictions, given several points on a plane it is usually possible to "connect-the-dots" in such a way as to yield a continuous curve.[11] Yet the mutual positioning of the points demands certain properties be met by this curve; while there are an infinite number of curves that satisfy the given criteria, there are just as many curves that do not.

Just a few sentences later, to further explicate his understanding of "happen," Erasmas introduces the mathematical idea of extension fields which can, with some poetic license, be thought of as the algebraic analog of continuity. Erasmas observes, "See, it's not just about what is *possible*—since anything is possible in Hemn space—but what is *compossible*, meaning all the other things that would have to be true in that universe, to have a block of ice in a star" (682). Likewise, studies of extension fields center around understanding the necessary conditions that must be met if a given field, say the rational numbers, is to be extended in such a way as to include an irrational number, say $\sqrt{2}$.[12] In order for an event to occur, Erasmas indicates, other supporting events must also occur of necessity, just as extension fields require the inclusion of other numbers beside the explicit desiderata.

Such mathematical analysis is useful for understanding which permutations of physical objects and laws do and do not allow for coherent universes in the world of *Anathem*, but it does not resolve other issues of mathematics and intercosmic communication. To return to the first calca, Erasmas observes to Dath, following the discussion of geometric omnipresence,

> No matter who is discussing these proofs, in what age, whether they are speaking of sheet cake or pastureland, they always arrive at the same answer. These truths seem to come out of another world or plane of existence. It's hard not to believe that this other world really exists in some sense—not just in our imaginations! (149)

The Procians of the novel are quick to ridicule the naive Platonism espoused in the last sentence of this quotation. As one of the leading Procians claims, "Language is nothing more than a stream of symbols that are perfectly meaningless until we associate them, in our minds, *with* meaning: a process of acculturation" (659), implicitly including the discourse of mathematics under the activity of language. Both philosophies are coherent, when considered in their own right, and both can be advocated by a commonsense mentality, albeit mentalities with different bents to them.

The crux of the matter centers around the following question: why do the geometric systems of Arbre and the geometric systems of the aliens align? More generally, why does mathematics appear to be universal? The Procian/

Fictionalist philosophy has a hard time answering this question, but the Halikaarnian/Platonist solution, while more immediately apparent, seems intuitively absurd. The reconciliation of these two discourses derives from the theory of Platonic cosmi. As argued in the third calca, there is no particular reason why there should be only two cosmi—the physical world, and "ideal" world, with the latter impinging on the former, but remaining unaffected by it. Rather, as long as certain structural rules are obeyed, it is logically coherent to speak of arbitrarily many cosmi, each "ideal" with respect to some cosmi, and physical with respect to others, with information flowing from the "more" ideal cosmi to the "less" ideal cosmi. All that is required for this coherency to hold is that the information not loop back around; that is, the flow of information from a given cosmos cannot return back to that same cosmos. Thus, it is proposed that both the cosmos of Arbre and the cosmos of the aliens are recipients of the same flow of information from a more ideal universe, which—considered with respect to itself—appears just as physical as Arbre, but when considered from the perspective of Arbre, appears transcendent and absolute. Thus, when Erasmas speaks of the universality of mathematics, he is referring to mathematics as it holds in a higher, more ideal universe, an explanation that works to maintain the coherency of Halikaarnian models of mathematics, while simultaneously enabling the coherency of Procian models of mathematics. That is, under this model, mathematical statements refer to true, existing things qua Platonist philosophies of mathematics, but these things exist not through a quasi-mystical self-supporting ontology but as a result of a language game played by beings in a higher universe. Thus, it is observed that "If it is true that the [aliens] share the Adrakhonic Theorem and other such theorical concepts with us […] those might be nothing more than attractors in the feedback system we have been describing" (738), a statement both the Procians and the Halikaarnians of the novel have little trouble agreeing with.

The novel engages in a bit of handwaving in explaining *how* this information percolates down to lower universes, but the suggestion that it *does* percolate in such a way enables a interesting and theoretically coherent resolution to the Platonist/Fictionalist impasse. Of course, as a piece of rigorous philosophy, the theories set forth in *Anathem* do not hold up when considered up close and in detail. For example, while postulating a higher cosmos that is simultaneously physical and transcendent can solve the local problem of how mathematics works in the present cosmos, it does not explain how information can flow in a global context; that is, the novel seems to imply a infinite regression of potential cosmi, each higher up the "flow" than the one before

it, but in order for the flow to reach a present universe, it seems necessary that there be some "first" cosmos—how these two observations are to be reconciled under this model does not seem to me to be immediately obvious. However, as mentioned at the beginning of this article, it is important to treat the ideas of *Anathem* not as a making up a scientific treatise but a "likely story," and as such the novel is more concerned with presenting ideas that are interesting and possibly true than ideas that may not be all that interesting but are certainly true. At any rate, the ideas of *Anathem* are certainly interesting and thought-provoking in their own right, and may, with time, come to provide a conceptual framework for developing a resolution between Platonist and Fictionalist modes of thought, enabling further innovations in the discourse of mathematical philosophy

Notes

1 The sometimes alienating effect of these neologisms in the novel has prompted popular Internet figure and webcomic writer Randall Munroe to declaim, "Except for anything by Lewis Carroll or Tolkien you get five made-up words per story. I'm looking at you, *Anathem*" (xkcd.com/483 [alt-text])
2 "In Mathic architecture, the tallest structure in a concent, typically the clock tower" (946).
3 I use "infinite" here in a loose, metaphorical sense. A more accurate description of a circle, albeit still an informal one, would be as the limit of a series of polygons whose number of sides increased to arbitrarily large numbers (i.e. hexagon, heptagon, octagon, nonagon . . . 100-gon . . etc. etc.).
4 In his commentaries on *Anathem*, Stephenson is quite insistent that the Præsidium reflects his interest in the 10,000-year clock project of the Long Now Foundation; however, it is difficult to believe that a writer as thoroughly familiar with the history of Western philosophy as Stephenson seems to be would not be conscious of the Heideggerian implications of the tower.
5 "At many concents, the large, centrally located building that houses the clock and that serves as the venue for auts and other gatherings of the entire population" (944–5).
6 In this context, equivalent to "Euclidean."
7 In this context, equivalent to the Pythagorean Theorem.
8 "An explanation, definition, or lesson that is instrumental in developing some larger theme, but that has been moved aside from the main body of the dialog and encapsulated in a footnote or appendix" (935).
9 The problem of the One and the Many has been around since at least the time of Plato. While it has appeared in many forms and variations, the general idea of the problem is to make sense of the way in which objects that appear to be singular can often, in fact, be made up of many constituent elements. It is closely related to

the classical attempt to find the fundamental principle of unity, that "Oneness" which alone cannot be reduced any further. It is interesting to note that the German mathematician Georg Cantor thought that set theory could partially solve this problem, famously defining a set as "a many that can be thought of as a one."

10 Indeed, argumentation from aesthetics is found throughout mathematical literature. Typically, arguments from deductive logic are used to establish theorems within the context of axiomatic systems; the justification for the axiomatic systems, however, is more often aesthetic than anything else. Consider, for example, the following justification for the ostensibly excessive use of the D4 permutation table for illustrative exercises, taken from John B. Fraleigh's *A First Course in Abstract Algebra*, a standard upper-division algebra text: "Look at the lovely symmetries in that table![…] Look at the lovely symmetries in that diagram!" (81). Clearly, Fraleigh to some extent at least, admits the aesthetic as a method of valid argumentation.

11 Of course, exceptions to this rule exist. Of these exceptions, perhaps the most famous is Dirichlet's function, which, in one version, can be described as follows: if x is rational, let $y = 1$; if x is irrational, let $y = 0$. Since between any two rational points there exists an irrational point, and between any two irrational points there exists a rational point, the function quickly "jumps" between 0 and 1, far too quickly to allow for any kind of continuity between points. While defined everywhere on the real number line, the function is therefore continuous nowhere. However, the existence of "pathological" cases like Dirichlet's function does not problematize the argument of *Anathem*; rather, they represent "incoherent" cosmi, cosmi in which the proposed worldtrack could not have happened.

12 For example, although there is not enough space here to go into the details of the proof, in order to extend the field of rational numbers by $\sqrt{2}$, and have the resulting set and residual operations remain a field, it is necessary to include all products $a \times \sqrt{2}$, where a is a rational number, in the field as well.

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